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BY A MULTIPLY EXPOSED PICTURE

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# IMAGE RESTORATIONS CONSTRAINED WITH A MULTIPLY-EXPOSED PICTURE[*]

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## Abstract

There are a number of possible industrial and scientific applications of nanosecond cineradiographs. While the technology exists to produce closely spaced pulses of X rays for this application, the quality of the time-resolved radiographs is severely limited. The limitations arise from the necessity of using a fluorescent screen to convert the transmitted X rays to light and then using electro-optical imaging systems to gate and to record the images with conventional high-speed cameras. It has been proposed that in addition to the time-resolved images, a conventional, multiply-exposed radiograph be obtained. This paper uses simulations to demonstrate that the additional information supplied by the multiply-exposed radiograph can be used to improve the quality of digital image restorations of the time-resolved pictures over what could be achieved with the degraded images alone. Because of the need for image registration and rubber sheet transformations, this problem is one which can best be solved on a digital, as opposed to an optical, computer.

## Introduction

At the Los Alamos Scientific Laboratory (LASL) novel image processing techniques are often developed to solve problems arising in the unique experiments being carried out at that laboratory. Recent studies related to improving a LASL radiographic facility raised an interesting image processing question. Could a multiply-exposed, but otherwise undegraded, image be used to improve the quality of digital restorations of degraded versions of the individual scenes in the multiple image? Since the multiple exposure is additional information, the intuitive answer is "yes!" This paper demonstrates this point. Below we describe an algorithm to use the additional information in the multiple exposure to improve the restorations beyond those possible with only the degraded images and knowledge of the degrading function and image statistics.

One rarely finds an imaging experiment in which separate low-quality pictures are obtained along with a high-quality multiple exposure. To justify this study, it is necessary to describe the radiographic experiment which led to it. PHERMEX is a 30 MeV electron accelerator at LASL used for flash (i.e., stop-action) radiography of rapidly moving objects. Advances in accelerator technology have made it possible to produce numbers of intense pulses of X rays which are closely spaced in time. With this capability ultra-high speed cineradiography will be possible. It will be possible to produce at least three 40 nsec bursts of radiation over a time span of between 4 and 1000 μsec. Each radiation pulse would have an intensity of 100 R.

In a single-pulse flash radiography the usual image recording medium is film contained in special cassettes. Because of the size and weight of these cassettes, the film cannot be changed between the pulses. To take advantage of the multiple pulse capability being developed for the PHERMEX facility, it was necessary to develop techniques for recording numbers of X-ray images closely spaced in time. The approach chosen was to convert the X-ray images into light images using a fluorescent screen. Light from the fluorescent screen is then recorded using a conventional high-speed, electronically intensified framing camera. Since the electro-optical camera imagery was poorer than conventional radiographs, digital image restoration was needed. The penetrating nature of X rays makes it possible to place a conventional X-ray film cassette between the object and the fluorescent screen. The image recorded on such a film would be the superposition of radiographs from each of the radiation pulses. Each component image would have the clarity of conventional PHERMEX radiography. The possibility of obtaining this additional image for each series of time-resolved radiographs motivated this study.

In this report we will present a model of the radiographic experiment from which simulations are derived. The simulations use conventional photographs. An algorithm for restoration of the degraded images constrained by the multiply-exposed image will be derived. Experimental restorations of the simulated imagery will be presented. The work demonstrates the advantage of using the multiple-exposure in the restoration scheme.

## Image Model and Simulations

The degraded, time-resolved images are represented as the convolution of the undegraded images with a pointspread function. The noise is considered to be additive and signal-independent. Using matrix notation[1] the degraded image is given by the expression

$$g_i = H f_i + n_0 \qquad i = 1,2,\ldots N \qquad (1)$$

where $g_i$ represents the degraded image and $f_i$ represents the input image. H is obtained from the point spread function and n is the noise term. The multiple exposure is given by the expression

$$f_m = \sum_{i=1}^{N} f_i \qquad (2)$$

For ordinary light photography the quantities $f_i$ in Eq. (2) would be light intensities. For high-energy X rays recorded without fluorescent intensification screens the film density of the radiograph is proportional to the incident radiation intensity.[2] For X rays the $f_i$'s in Eq. (2) could be film densities.

We used three photographs of similar objects with large regions of relatively high density to simulate the $f_i$'s. Figure 1 shows one of the digitized images. Figure 2 is the simulated multiple-exposure image. The simulated pointspread function was a Gaussian with a standard deviation of 1.4 pixels. The noise function was a white, zero-mean Gaussian with a standard deviation of 0.04 density units. The signal-to-noise ratio was about 20 db. Figure 3 shows the simulation of one of the degraded images.
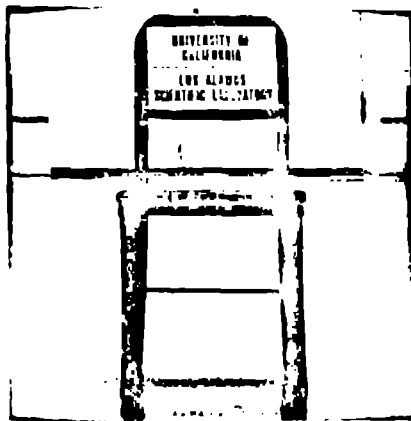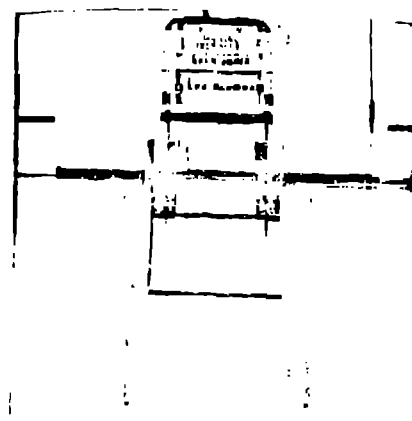


Fig. 1. Original Image



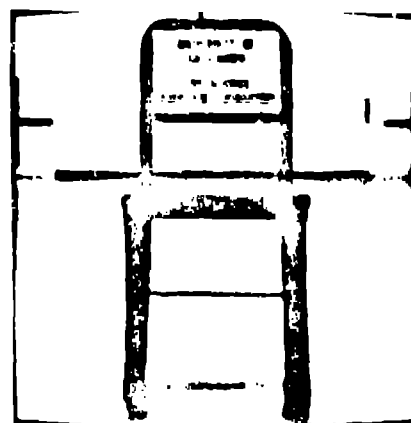Fig. 2. Simulated Multiple-exposure image (scaled)



Fig. 3. Simulated degraded image

## RESTORATION ALGORITHM

One approach to restoring the $g_i$'s is to seek an estimate $\hat{f}_i$ which will minimize

$$\sum_{i=1}^{N} (g_i - H\hat{f}_i)^T(g_i - H\hat{f}_i) \tag{3}$$

subject to the constraint

$$f_m = \sum_{i=1}^{N} \hat{f}_i \tag{4}$$

In these equations N is the number of degraded images. The solution is obtained using Lagrange multipliers. Minimize the quantity

$$s = \sum_{i=1}^{N} (g_i - H\hat{f}_i)^T(g_i - H\hat{f}_i) + \lambda^T(f_m - \sum_{i=1}^{N} \hat{f}_i) \tag{5}$$

with respect to the $\hat{f}_i$'s. $\lambda^T$ is a row vector of N Lagrange multipliers. After carrying out the minimization of s and assuming that $H^{-1}$ exists, the solution is

$$f_i = \frac{N-1}{N} H^{-1}g_i + \frac{1}{N}\left[f_M - \sum_{j \neq i=1}^{N} H^{-1}g_j\right] \tag{6}$$

While the existence of $H^{-1}$ could be questioned, Eq. (6) is very easy to derive and analyze. This expression for $\hat{f}_i$ is the essence of this paper. The solution is very satisfying intuitively. If N were 1, then the undegraded "multiple" image is the solution. As N increases, our ability to unscramble the multiple image decreases. Therefore, we rely more on the straightforward restoration for the solution. One would expect that in regions where the restorations of the other images are poor, residuals will remain in the second term. These residuals appear as "ghosts" in the constrained restorations. These are particularly evident in high-contrast regions of the images. Ghosts will be unavoidable if imperfect global processing is used.

The following derivations show that the form of Eq. (6) holds for other restoration schemes. Let $\hat{f}_i$ be a linear combination of some arbitrary restoration $\mathscr{R}(g_i)$ and the difference $[f_m - \sum_{j \neq i} \mathscr{R}(g_j)]$. Let Eq. (4) be a constraint.

$$\hat{f}_i = \alpha \mathscr{R}(g_i) + \beta\left[f_M - \sum_{j \neq i} \mathscr{R}(g_j)\right] \tag{7}$$

Sum the $\hat{f}_i$'s to obtain

$$f_M = \sum_{i=1}^{N} \hat{f}_i = \alpha \sum_{i=1}^{N} \mathscr{R}(g_i) + \beta N f_M - \beta(N-1)\sum_{i=1}^{N} \mathscr{R}(g_i) \tag{8}$$

consequently, $\beta = 1/N$ and $\alpha = (N-1)/N$.

Now consider the restoration which minimizes the expected value of the total error between the $f_i$'s and the $\hat{f}_i$'s.

Minimize:
$$E\left\{\sum_{i=1}^{N} [f_i - \hat{f}_i]^T[f_i - \hat{f}_i]\right\} \tag{9a}$$

where $E[\cdots]$ is the expected value. Substituting Eq. (7) into Eq. (9a) and assuming $\mathscr{R}(g_i) = Wg_i$ where W is a linear operator one obtains the expression

Minimize:
$$E\left\{\left|f_i - \frac{N-1}{N}Wg_i - \frac{1}{N}f_m + \frac{1}{N}\sum_{j\neq i=1}^{N}Wg_j\right|^2\right\}. \tag{9b}$$

If one expands the terms inside the summation, sums each term, differentiates with respect to W, and sets the result equal to zero. Eq. (9b) becomes

$$E\left\{-\frac{2(N-1)}{N}\sum_{i=1}^{N}f_ig_i^T + \frac{2}{N}\sum_{i=1}^{N}f_i\sum_{j\neq 1}g_i^T\right.$$

$$+ \frac{2(N-1)^2}{N^2}\sum_{i=1}^{N}Wg_ig_i^T + \frac{2(N-1)}{N^2}f_M\sum_{i=1}^{N}g_i$$

$$-\frac{4(N-1)}{N^2}W\sum_{i=1}^{N}g_i\sum_{j\neq i}g_i - \frac{2(N-1)}{N^2}f_M\sum_{i=1}^{N}g_i$$

$$\left.+ \frac{2}{N^2}W\sum_{i=1}^{N}\left(\sum_{j\neq i}g_j\right)\left(\sum_{j\neq i}g_j^T\right)\right\} = 0 \tag{10}$$

Multiplying Eq. (10) by N/2, factoring out W, and using the short hand $g_m = \sum_{i=1}^{N}g_i$ one finds

$$E\left\{W\left[\sum_{i=1}^{N}g_ig_i^T - \frac{1}{N}g_mg_m^T\right]\right\} = E\left\{\sum_{i=1}^{N}f_ig_i^T - \frac{1}{N}f_mg_m^T\right\} \tag{11}$$

Using Eq. (1) and assuming $E\{f_n^T\} = 0$, then $E\{f_ig_i^T\} = E\{f_if_i^TH^T\}$. With the usual assumption that the images are independent and are drawn from a zero mean ensemble, then $E\{f_if_j^T\} = 0$. If ergodicity holds, $E\{f_if_i^T\} = E\{f_jf_j^T\}$. Therefore, subscripts can be dropped. W, the restoration filter that produces the minimum mean square error, is given by

$$W = \frac{E\{ff^T\}H^T}{H\,E\{ff^T\}H^T + E\{nn^T\}} \tag{12}$$

This is the standard Wiener filter often used in image processing. The filter which minimizes the mean square error for the N images with the constraint is the filter which minimizes the mean square error for each image separately.

## RESULTS OF IMAGE PROCESSING

We restored three simulated, degraded images using both conventional Wiener restorations and the approach given in Eq. (7). Table 1 gives the mean square error between the two restorations and the original images. Figures 4-9 show restoration pairs for the three simulations. The improvement in the mean square error and the superiority of the constrained restoration needs no further elaboration.

Table 1.

Comparison of Mean Square Errors
for Wiener and Constrained Restoration

$$MSE = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| f_{ij} - \hat{f}_{ij} \right|^2 .$$

| Image Number | Mean Square Error | |
|---|---|---|
| | Wiener | Constrained |
| 1 | $9.6 \times 10^{-3}$ | $5.0 \times 10^{-3}$ |
| 2 | $8.0 \times 10^{-3}$ | $4.6 \times 10^{-3}$ |
| 3 | $7.2 \times 10^{-3}$ | $3.8 \times 10^{-3}$ |

**Further** study will continue into the effects of more severe degradations and more severe noise. The question of a different degradation of the multiple image is also being studied. At this point we have shown that the additional information provided by the multiple image can be used to improve restorations of the individual, time-resolved images.
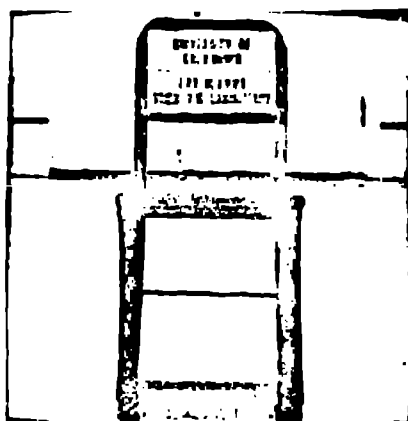

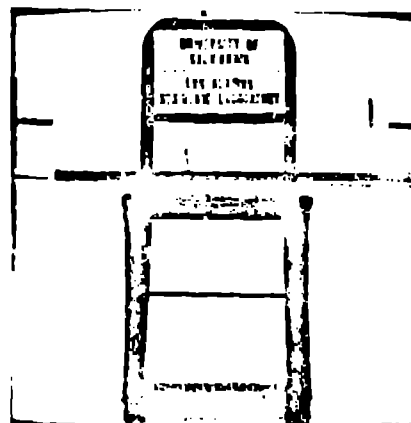
Fig. 4. Image 1 restored using Wiener Filter
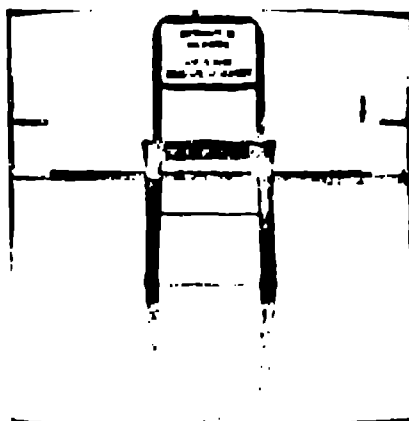


Fig. 5. Contrained restoration of Image 1
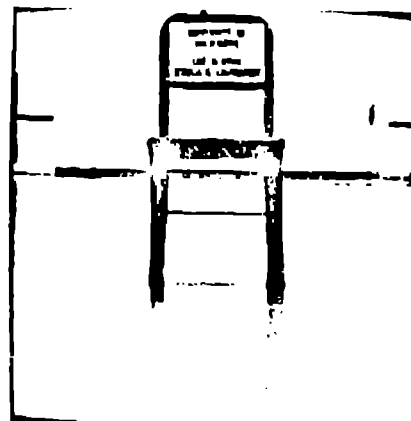


Fig. 6. Image 2 restored using Wiener Filter
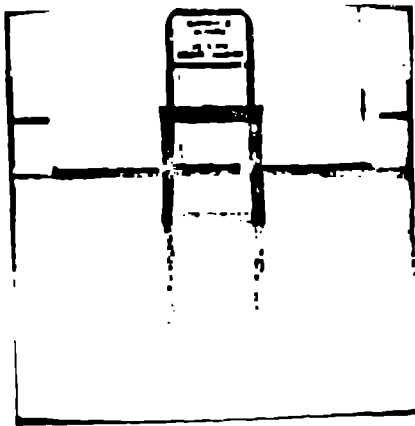


Fig. 7. Contrained restoration of Image 2
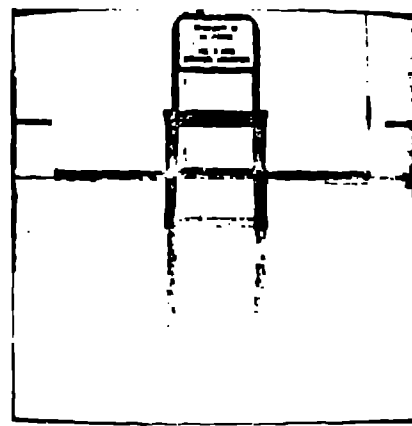
Fig. 8. Image 3 restored using Wiener Filter



Fig. 9. Constrained restoration of Image 3

## References

1. Andrews, H. C. and Hunt, B. R., Digital Image Restoration, Prentice Hall, Inc. 1977, Chapter 3.
2. Herz, R. H., The Photographic Action of Ionizing Radiations, Wiley-Interscience 1969.